

Revisiting Axial-Vector Meson Mixing

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Abstract

Various phenomenological studies indicate that the mixing angle θ_{K_1} of K_{1A} and K_{1B} , the strange partners of the axial-vector mesons $a_1(1260)$ and $b_1(1235)$, respectively, lies in the vicinity of 35° or 55° , but whether this angle is larger or smaller than 45° still remains controversial. When the $f_1(1285)$ - $f_1(1420)$ mixing angle θ_{3P_1} and the $h_1(1170)$ - $h_1(1380)$ mixing angle θ_{1P_1} are determined from the mass relations, they depend on the masses of K_{1A} and K_{1B} , which in turn depend on the mixing angle θ_{K_1} . We show that the approximate decoupling of the light $q\bar{q}$ state from the heavier $s\bar{s}$ state, which is empirically valid for vector, tensor and 3^{--} mesons, when applied to isoscalar axial-vector mesons, will enable us to discriminate different solutions of θ_{3P_1} and θ_{1P_1} and pick up $\theta_{K_1} \sim 35^\circ$. Indeed, for $\theta_{K_1} \sim 55^\circ$, the predicted θ_{1P_1} disagrees sharply with the recent lattice calculation and the implied large $s\bar{s}$ content of $h_1(1170)$ and $q\bar{q}$ component of $h_1(1380)$ cannot explain the observation of their strong decays. We conclude that θ_{K_1} is smaller than 45° .

I. INTRODUCTION

The mixing of self-conjugate mesons in a generalized QCD-like theory was recently discussed in [1] with emphasis on the role of decoupling. The mixing of the flavor-SU(3) singlet and octet states of vector and tensor mesons to form mass eigenstates is of fundamental importance in hadronic physics. In the case of the vector mesons, the physical ω is mostly comprised of the isospin-singlet combination $(u\bar{u}+d\bar{d})/\sqrt{2}$, while ϕ is mostly an $s\bar{s}$ state. In a modern context, some insight into this comes from the Appelquist-Carazzone decoupling theorem [3], according to which, in a vectorial theory, as the mass of a particle gets large compared with a relevant scale, say, $\Lambda_{QCD} \simeq 300$ MeV, one can integrate this particle out and define a low-energy effective field theory applicable below this scale. Evidently, even though m_s is not $\gg \Lambda_{QCD}$, there is still a nearly complete decoupling. A similar situation of near-ideal mixing occurs for the $J^{PC} = 2^{++}$ tensor mesons $f_2(1275)$, $f'_2(1525)$ and the $J^{PC} = 3^{--}$ mesons $\omega_3(1670)$, $\phi_3(1850)$ and can also be understood in terms of approximate decoupling of the light $u\bar{u} + d\bar{d}$ state from the heavier $s\bar{s}$ state.

There exist two different types of nonets for $J^P = 1^+$ axial-vector mesons which arise as orbitally excited quark-antiquark bound states: 1^3P_1 and 1^1P_1 . These two nonets have different C quantum numbers for their respective neutral mesons, namely $C = +$ and $C = -$. The non-strange axial vector mesons, for example, the neutral $a_1(1260)$ and $b_1(1235)$ cannot mix because of their opposite C -parities. In contrast, the mesons K_{1A} and K_{1B} , the strange partners of $a_1(1260)$ and $b_1(1235)$, respectively, do mix to form corresponding physical mass eigenstates $K_1(1270)$ and $K_1(1400)$. This complicates the analysis of the mixings of the SU(3)-singlet and SU(3)-octet mesons in the 1^3P_1 and 1^1P_1 nonets. Various phenomenological studies indicate that the K_{1A} - K_{1B} mixing angle θ_{K_1} is around either 35° or 55° , but there is no consensus as to whether this angle is greater or less than 45° .

In the present work, we shall show that when applying the approximate decoupling of the light $q\bar{q}$ state from the heavier $s\bar{s}$ state to the axial-vector mesons, we are able to pin down the mixing angle θ_{K_1} . This is based on the observation that when the $f_1(1285)$ - $f_1(1420)$ mixing angle θ_{3P_1} and the $h_1(1170)$ - $h_1(1380)$ mixing angle θ_{1P_1} are determined from the mass relations, they depend on the masses of K_{1A} and K_{1B} , which in turn depend on θ_{K_1} . Nearly complete decoupling will allow us to discriminate different solutions of θ_{3P_1} and θ_{1P_1} and pick up the right mixing angle θ_{K_1} .

The layout of the present paper is organized as follows. We first recapitulate in Sec. II the main results derived in [1] for isoscalar meson mixing. Then we proceed to consider the mixing of axial-vector mesons in Sec. III and discuss the physical implications in Sec. IV. We give the conclusions in Sec. V.

II. SOME RELATIONS FOR MESON MIXING

In this section we recapitulate some results in [1] for meson mixing, where we have considered an SU(N_c) QCD-like theory with $\ell = N_f - 1$ massless or light quarks q_i , $i = 1, \dots, \ell$, and one quark Q of substantial mass m_Q . For $N_c = 3$ and $\ell = 2$, this theory is a rough approximation to real QCD, since the current-quark masses of the u and d quarks satisfy $m_u, m_d \ll \Lambda_{QCD}$, while $m_s \sim 100$ MeV is smaller than, but comparable to Λ_{QCD} , and one can focus on the effective

QCD theory with the heavy quarks c , b , t integrated out. Considering the mass-squared matrix M^2 in the basis of the SU(3) singlet and octet flavor eigenstates $|V_1\rangle$ and $|V_8\rangle$, respectively, with $|V_1\rangle = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $|V_8\rangle = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, we can write it as the real, symmetric matrix¹

$$M^2 = \begin{pmatrix} m_1^2 & \delta \\ \delta & m_8^2 \end{pmatrix}. \quad (2.1)$$

This mass matrix is diagonalized according to

$$R(\theta)M^2R(\theta)^{-1} = M_{diag}^2. \quad (2.2)$$

with

$$M_{diag}^2 = \begin{pmatrix} m_L^2 & 0 \\ 0 & m_H^2 \end{pmatrix}, \quad R(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (2.3)$$

The eigenvalues of M^2 are given by

$$m_{H,L}^2 = \frac{1}{2} \left[m_8^2 + m_1^2 \pm \sqrt{(m_8^2 - m_1^2)^2 + 4\delta^2} \right]. \quad (2.4)$$

One can work backward from the observed masses and mixing angle to determine δ . The mass squared matrix then becomes

$$M^2 = \begin{pmatrix} m_L^2 + m_H^2 - m_8^2 & -\left(m_8^2(m_L^2 + m_H^2 - m_8^2) - m_L^2 m_H^2\right)^{1/2} \\ -\left(m_8^2(m_L^2 + m_H^2 - m_8^2) - m_L^2 m_H^2\right)^{1/2} & m_8^2 \end{pmatrix}, \quad (2.5)$$

where the mass squared of the SU(3)-octet m_8^2 can be determined from the Gell-Mann Okubo mass relation [4]. The mixing angle can be expressed in several different but equivalent forms:

$$\tan 2\theta = -\frac{2\delta}{m_8^2 - m_1^2}, \quad \cos 2\theta = \frac{m_8^2 - m_1^2}{m_H^2 - m_L^2}, \quad (2.6)$$

$$\tan \theta = \frac{m_8^2 - m_H^2}{\delta}, \quad \cot \theta = -\frac{m_8^2 - m_L^2}{\delta}, \quad (2.7)$$

$$\tan^2 \theta = \frac{m_H^2 - m_8^2}{m_8^2 - m_L^2}, \quad \cos^2 \theta = \frac{m_8^2 - m_L^2}{m_H^2 - m_L^2}. \quad (2.8)$$

Eqs. (2.6) and (2.7) have the advantage that the magnitude and the sign of the mixing angle are fixed simultaneously.

Applying the Applequist-Carazzone decoupling theorem [3], we infer that when m_s is treated as a variable and increases past Λ_{QCD} , it is possible to define an effective low-energy theory with the s quark integrated out. Hence, the mixing of the meson flavor eigenstates must be such as to produce a mass eigenstate composed of the light u and d quarks, and an orthogonal mass eigenstate composed only of the s quark. The decoupling angle is given by

$$\theta_{dec.} = \arctan\left(\frac{1}{\sqrt{2}}\right) = 35.26^\circ. \quad (2.9)$$

¹ Here and below, we follow the common practice of using the squared masses of the mesons rather than the masses themselves, because for bosons it is the squared mass that appear in effective Lagrangians.

It turns out that the physical mixing angles 39.0° , 29.5° and 32.0° , respectively, for $J^{PC} = 1^{--}$ (vector), 2^{++} (tensor) and 3^{--} mesons [2] are indeed close to the ideal one. Especially, the vector mixing angle is in good agreement with the value $\theta_{V,ph} = (38.58 \pm 0.09)^\circ$ obtained from a recent global fit by KLOE [5]. *A priori*, one does not expect a large decoupling effect because m_s is not large compared to Λ_{QCD} , $m_s/\Lambda_{QCD} \simeq 1/3$. Indeed, one of the most intriguing aspects of ω - ϕ mixing is how close this is to the decoupling limit even though m_s/Λ_{QCD} is not $\gg 1$.

III. MIXING OF AXIAL-VECTOR MESONS

In the quark model, two nonets of $J^P = 1^+$ axial-vector mesons are expected as the orbital excitation of the $q\bar{q}$ system. In terms of the spectroscopic notation $^{2S+1}L_J$, there are two types of P -wave axial-vector mesons, namely, 3P_1 and 1P_1 . These two nonets have distinctive C quantum numbers for the corresponding neutral mesons, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$ and K_{1A} , while the 1^{+-} nonet contains $b_1(1235)$, $h_1(1170)$, $h_1(1380)$ and K_{1B} . The non-strange axial vector mesons, for example, the neutral $a_1(1260)$ and $b_1(1235)$ cannot have mixing because of the opposite C -parities. On the contrary, K_{1A} and K_{1B} are not the physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ and they are mixed together due to the strange and non-strange light quark mass difference. Following the common convention we write²

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin \theta_{K_1} & \cos \theta_{K_1} \\ \cos \theta_{K_1} & -\sin \theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}. \quad (3.1)$$

There exist several estimations on the mixing angle θ_{K_1} in the literature. From the early experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki found two possible solutions $\theta_{K_1} \approx 33^\circ$ and 57° [12]. A similar constraint $35^\circ \lesssim \theta_{K_1} \lesssim 55^\circ$ was obtained in Ref. [13] based solely on two parameters: the mass difference between the $a_1(1260)$ and $b_1(1235)$ mesons and the ratio of the constituent quark masses. An analysis of $\tau \rightarrow K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays also yielded the mixing angle to be $\approx 37^\circ$ or 58° [9]. Another determination of θ_{K_1} comes from the $f_1(1285)$ - $f_1(1420)$ mixing angle θ_{3P_1} to be introduced shortly below which can be reliably estimated from the analysis of the radiative decays $f_1(1285) \rightarrow \phi\gamma, \rho^0\gamma$ [14]. A recent

² The sign of the mixing angle θ_{K_1} and the relative signs of the decay constants as well as form factors for K_{1A} and K_{1B} were often very confusing in the literature. As stressed in Ref. [6], the sign of θ_{K_1} is intimately related to the relative sign of the K_{1A} and K_{1B} states which can be arbitrarily assigned. This sign ambiguity can be removed by fixing the relative sign of the decay constants of K_{1A} and K_{1B} . For example, in the covariant light-front quark model [7] and in pQCD [8], the decay constants $f_{K_{1A}}$ and $f_{K_{1B}}$ are of opposite sign, while the $D(B) \rightarrow K_{1A}$ and $D(B) \rightarrow K_{1B}$ transition form factors have the same signs. Then θ_{K_1} is positive as the negative one is ruled out by the data of $D^+ \rightarrow \bar{K}_1^0(1270)\pi^+$, $D^0 \rightarrow K_1^-(1270)\pi^+$ [9, 10] and also by the measurements of $B \rightarrow K_1(1270)\gamma$ and $B \rightarrow K_1(1400)\gamma$ [11]. In this work, we shall choose the convention for decay constants in such a way that θ_{K_1} is positive. Therefore, the values of θ_{K_1} cited from various references below are always positive in our convention. Note that for the antiparticle states $\bar{K}_1(1270)$, $\bar{K}_1(1400)$, \bar{K}_{1A} and \bar{K}_{1B} , the mixing angle is of opposite sign to that defined in Eq. (3.1).

updated analysis yields $\theta_{3P_1} = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$ [15].³ As we shall see below, the mixing angle θ_{3P_1} is correlated to θ_{K_1} . The corresponding θ_{K_1} is found to be $(31.7^{+2.8}_{-2.5})^\circ$ or $(56.3^{+3.9}_{-4.1})^\circ$. Therefore, all the analyses yield a mixing angle θ_{K_1} in the vicinity of either 35° or 55° .

However, there is no consensus as to whether θ_{K_1} is greater or less than 45° . It was found in the non-relativistic quark model that $m_{K_{1A}}^2 < m_{K_{1B}}^2$ [16–18] and hence θ_{K_1} is larger than 45° (see Eq. (3.7) below).⁴ Interestingly, θ_{K_1} turned out to be of order 34° in the relativized quark model of [19]. Based on the covariant light-front model [7], the value of 51° was found by the analysis of [20]. From the study of $B \rightarrow K_1(1270)\gamma$ and $\tau \rightarrow K_1(1270)\nu_\tau$ within the framework of light-cone QCD sum rules, Hatanaka and Yang advocated that $\theta_{K_1} = (34 \pm 13)^\circ$ [21]. In short, there is a variety of different values of the mixing angle cited in the literature. It is the purpose of this work to pin down θ_{K_1} .

We next consider the mixing of the isosinglet 3P_1 states, $f_1(1285)$ and $f_1(1420)$, and the 1P_1 states, $h_1(1170)$ and $h_1(1380)$:

$$\begin{pmatrix} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{3P_1} & \sin \theta_{3P_1} \\ -\sin \theta_{3P_1} & \cos \theta_{3P_1} \end{pmatrix} \begin{pmatrix} |f_1\rangle \\ |f_8\rangle \end{pmatrix}, \quad (3.2)$$

and

$$\begin{pmatrix} |h_1(1170)\rangle \\ |h_1(1380)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{1P_1} & \sin \theta_{1P_1} \\ -\sin \theta_{1P_1} & \cos \theta_{1P_1} \end{pmatrix} \begin{pmatrix} |h_1\rangle \\ |h_8\rangle \end{pmatrix}, \quad (3.3)$$

where $f_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, $f_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, and likewise for h_1 and h_8 . Using the squared mass matrix Eq. (2.5) with some appropriate replacements such as $m_L = m_{f_1(1285)}$, $m_H = m_{f_1(1420)}$ etc. for 3P_1 states and $m_L = m_{h_1(1170)}$, $m_H = m_{h_1(1380)}$ etc. for 1P_1 states, and applying the Gell-Mann Okubo relations for the mass squared of the octet states

$$\begin{aligned} m_8^2(^3P_1) &\equiv m_{3P_1}^2 = \frac{1}{3}(4m_{K_{1A}}^2 - m_{a_1}^2), \\ m_8^2(^1P_1) &\equiv m_{1P_1}^2 = \frac{1}{3}(4m_{K_{1B}}^2 - m_{b_1}^2), \end{aligned} \quad (3.4)$$

we obtain from Eqs. (2.7) and (2.8) that

$$\begin{aligned} \tan \theta_{3P_1} &= \frac{m_{3P_1}^2 - m_{f_1'}^2}{\sqrt{m_{3P_1}^2(m_{f_1}^2 + m_{f_1'}^2 - m_{3P_1}^2) - m_{f_1}^2 m_{f_1'}^2}}, \\ \tan \theta_{1P_1} &= \frac{m_{1P_1}^2 - m_{h_1'}^2}{\sqrt{m_{1P_1}^2(m_{h_1}^2 + m_{h_1'}^2 - m_{1P_1}^2) - m_{h_1}^2 m_{h_1'}^2}}, \end{aligned} \quad (3.5)$$

and

$$\begin{aligned} \tan^2 \theta_{3P_1} &= \frac{4m_{K_{1A}}^2 - m_{a_1}^2 - 3m_{f_1'}^2}{-4m_{K_{1A}}^2 + m_{a_1}^2 + 3m_{f_1}^2}, \\ \tan^2 \theta_{1P_1} &= \frac{4m_{K_{1B}}^2 - m_{b_1}^2 - 3m_{h_1'}^2}{-4m_{K_{1B}}^2 + m_{b_1}^2 + 3m_{h_1}^2}, \end{aligned} \quad (3.6)$$

³ From the same radiative decays, it was found $\theta_{3P_1} = (56^{+4}_{-5})^\circ$ in [14]. This has led some authors (e.g. [16]) to claim that $\theta_{K_1} \sim 59^\circ$. However, another solution, namely, $\theta_{3P_1} = (14.6^{+4}_{-5})^\circ$ corresponding to a smaller θ_{K_1} , was missed in [14].

⁴ As pointed out in [16], the solutions $\theta_{K_1} = (37.3 \pm 3.2)^\circ$ obtained in [17] and $(31 \pm 4)^\circ$ in [18] should be replaced by $\pi/2 - \theta_{K_1}$.

TABLE I: The values of the $f_1(1285)$ - $f_1(1420)$ and $h_1(1170)$ - $h_1(1380)$ mixing angles θ_{3P_1} and θ_{1P_1} , respectively, calculated using Eq. (3.5) for some representative K_{1A} - K_{1B} mixing angle θ_{K_1} .

| θ_{K_1} | 57° | 51° | 45° | 34° |
|-----------------|---------------|--------------|--------------|--------------|
| θ_{3P_1} | 52.0° | 45.1° | 37.9° | 23.1° |
| θ_{1P_1} | -17.5° | -9.1° | 14.4° | 28.0° |

where f_1 and f'_1 (h_1 and h'_1) are the short-handed notations for $f_1(1285)$ and $f_1(1420)$ ($h_1(1170)$ and $h_1(1380)$), respectively, and

$$\begin{aligned} m_{K_{1A}}^2 &= m_{K_1(1400)}^2 \cos^2 \theta_{K_1} + m_{K_1(1270)}^2 \sin^2 \theta_{K_1}, \\ m_{K_{1B}}^2 &= m_{K_1(1400)}^2 \sin^2 \theta_{K_1} + m_{K_1(1270)}^2 \cos^2 \theta_{K_1}. \end{aligned} \quad (3.7)$$

It is clear that the mixing angles θ_{3P_1} and θ_{1P_1} depend on the masses of K_{1A} and K_{1B} states, which in turn depend on the K_{1A} - K_{1B} mixing angle θ_{K_1} . Table I exhibits the values of θ_{3P_1} and θ_{1P_1} calculated using Eq. (3.5) for some representative values of θ_{K_1} . We see that while θ_{3P_1} is not far from the ideal mixing angle for $\theta_{K_1} < 50^\circ$, θ_{1P_1} is very sensitive to θ_{K_1} : Its deviation from exact decoupling increases with the increasing θ_{K_1} .

In the literature it is often to use Eq. (3.6) to determine the magnitude of the mixing angles θ_{3P_1} and θ_{1P_1} and the following relations

$$\tan \theta_{3P_1} = \frac{4m_{K_{1A}}^2 - m_{a_1}^2 - 3m_{f'_1}^2}{2\sqrt{2}(m_{a_1}^2 - m_{K_{1A}}^2)}, \quad \tan \theta_{1P_1} = \frac{4m_{K_{1B}}^2 - m_{b_1}^2 - 3m_{h'_1}^2}{2\sqrt{2}(m_{b_1}^2 - m_{K_{1B}}^2)} \quad (3.8)$$

to fix their signs (see e.g. [2, 22]). Consider the squared mass matrices

$$\begin{aligned} M^2(^3P_1) &= \frac{1}{3} \begin{pmatrix} 2m_{K_{1A}}^2 + m_{a_1}^2 + a_{1A} & -2\sqrt{2}(m_{K_{1A}}^2 - m_{a_1}^2) \\ -2\sqrt{2}(m_{K_{1A}}^2 - m_{a_1}^2) & 4m_{K_{1A}}^2 - m_{a_1}^2 \end{pmatrix}, \\ M^2(^1P_1) &= \frac{1}{3} \begin{pmatrix} 2m_{K_{1B}}^2 + m_{b_1}^2 + a_{1B} & -2\sqrt{2}(m_{K_{1B}}^2 - m_{b_1}^2) \\ -2\sqrt{2}(m_{K_{1B}}^2 - m_{b_1}^2) & 4m_{K_{1B}}^2 - m_{b_1}^2 \end{pmatrix}, \end{aligned} \quad (3.9)$$

for 3P_1 and 1P_1 states, respectively, where a_{1A} and a_{1B} are the parameters to be introduced below which will be set to zero for the moment. The above squared mass matrices can be derived from the non-relativistic quark model. Naively, if we substitute the above mass matrix elements in Eqs. (2.8) for $\tan^2 \theta$ and (2.7) for $\tan \theta$ and take $m_H^2 = m_{f'_1}^2$ ($m_{h'_1}^2$) and $m_L^2 = m_{f_1}^2$ ($m_{h_1}^2$) for the 3P_1 (1P_1) states, we will obtain Eqs. (3.6) and (3.8) for the mixing angles θ_{3P_1} and θ_{1P_1} . However, the mixing angles determined from these two equations are not the same in magnitude. For example, $|\theta_{3P_1}| = 23.1^\circ$ is deduced from the former and $\theta_{3P_1} = 10.5^\circ$ from the latter for $\theta_{K_1} = 34^\circ$. Since Eqs. (2.8) and (2.7) are equivalent, one may wonder why the resultant mixing angles are so different. This can be traced back to the fact that the mass eigenvalues m_H and m_L derived from the mass matrices (3.9) are not identical to the physical masses of $f_1(1420)$ and $f_1(1285)$, respectively, for 3P_1 states and $h_1(1380)$ and $h_1(1170)$ for 1P_1 states. That is, the mass matrices (3.9) do not lead

to Eqs. (3.6) and (3.8).⁵ Instead, they lead to the ideal mixing $\theta_{3P_1} = \theta_{1P_1} = 35.26^\circ$ (see [1] for a detailed discussion). In other words, the mass matrices $M^2(^3P_1)$ and $M^2(^1P_1)$ can be diagonalized by the orthogonal rotation matrix

$$R(\theta_{\text{dec.}}) = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \end{pmatrix}. \quad (3.10)$$

This result is unphysical, since it predicts that there is a complete decoupling of the s quark regardless of how small the nonzero mass difference $m_s - m_q$ is. This unphysical result shows that the initial quark model for the mass matrix is too simplistic. To remedy this defect, one takes account of the fact that there is a propagator correction (for both the kinetic and mass squared terms) in which the SU(3) flavor-singlet state $|V_1\rangle$ annihilates to an intermediate virtual purely gluonic state and then goes back to itself again [23]. This annihilation process denoted by a_{1A} and a_{1B} cannot occur for the flavor-SU(3) octet state, $|V_8\rangle$.

Since the squared mass matrix (3.9) derived from the non-relativistic quark model is only an approximation, in this work we should rely on the exact squared mass matrix given in (2.5) to get the mixing angles, namely, those shown in Table I. We would like to stress once again that Eqs. (3.5) and (3.6) all yield the same magnitude for θ_{3P_1} and θ_{1P_1} , but the former has the advantage that the sign and magnitude of the mixing angles can be fixed simultaneously.

IV. DISCUSSION

The values of the $f_1(1285)$ - $f_1(1420)$ and $h_1(1170)$ - $h_1(1380)$ mixing angles θ_{3P_1} and θ_{1P_1} , respectively, listed in Table I for some representative K_{1A} - K_{1B} mixing angle θ_{K_1} are the key results of this work. Although θ_{K_1} is unknown, we shall argue that the values of $\theta_{3P_1} \sim 23^\circ$ and $\theta_{1P_1} \sim 28^\circ$ as depicted in Table I are strongly preferred for the following reasons:

1. As discussed in Sec. II, nearly ideal mixing occurs for vector, tensor and 3^{--} mesons. Except for pseudoscalar mesons where the axial anomaly plays a unique role, this feature should also hold for axial-vector mesons. It is obvious from Table I that the mixing of isosinglet axial-vector mesons is close to the ideal one for $\theta_{K_1} \sim 34^\circ$ and far away from the decoupling limit (especially, $\theta_{1P_1} \sim -18^\circ$) when $\theta_{K_1} \sim 57^\circ$.
2. Since only the modes $h_1(1170) \rightarrow \rho\pi$ and $h_1(1380) \rightarrow K\bar{K}^*, \bar{K}K^*$ have been seen so far, this implies that the quark content is primarily $s\bar{s}$ for $h_1(1380)$ and $q\bar{q}$ for $h_1(1170)$. Likewise, $K^*\bar{K}$ and $K\bar{K}\pi$ are the dominant modes of $f_1(1420)$ whereas $f_1(1285)$ decays mainly to the $\eta\pi\pi$ and 4π states. These suggest that the quark content is primarily $s\bar{s}$ for $f_1(1420)$ and $q\bar{q}$ for $f_1(1285)$. Therefore, the observed strong decays of isoscalar axial-vector mesons suggest that their mixings are close to nearly decoupling. This in turn implies that $\theta_{K_1} \sim 34^\circ$ is much more favored. Indeed, if $\theta_{K_1} = 57^\circ$, we will have $\theta_{1P_1} = -18^\circ$ and $h_1(1170) = 0.60n\bar{n} - 0.80s\bar{s}$

⁵ It should be stressed that Eq. (3.8) (see also Eq. (14.9) of the Particle Data Group [2]) cannot be derived from any mass matrix. Unlike Eq. (3.5), it is *not* equivalent to Eq. (3.6).

and $h_1(1380) = 0.80n\bar{n} + 0.60s\bar{s}$ with $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. It is obvious that the large $s\bar{s}$ content of $h_1(1170)$ and $n\bar{n}$ content of $h_1(1380)$ cannot explain why only the strong decay modes $h_1(1170) \rightarrow \rho\pi$ and $h_1(1380) \rightarrow K\bar{K}^*, \bar{K}K^*$ have been seen thus far.

3. The $f_1(1285)$ - $f_1(1420)$ and $h_1(1170)$ - $h_1(1380)$ mixing angles α_{3P_1} and α_{1P_1} , respectively, in the flavor basis were recently calculated by the Hadron Spectrum Collaboration based on lattice QCD [24]. The results are $\alpha_{3P_1} = \pm(31 \pm 2)^\circ$ and $\alpha_{1P_1} = \pm(3 \pm 1)^\circ$. Since α is related to the singlet-octet mixing angle θ by the relation $\theta = 35.3^\circ + \alpha$,⁶ we have the two-fold solutions: $\theta_{3P_1} = (4.3 \pm 2)^\circ$ or $(66.3 \pm 2)^\circ$ and $\theta_{1P_1} = (32.3 \pm 1)^\circ$ or $(38.3 \pm 1)^\circ$. Evidently, the value of $\theta_{1P_1} \sim -18^\circ$ for $\theta_{K_1} \sim 57^\circ$ disagrees sharply with the lattice result. As for θ_{3P_1} , we recall that a study of the radiative decays $f_1(1285) \rightarrow \phi\gamma, \rho^0\gamma$ yields a direct determination of θ_{3P_1} to be $(19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$ [15]. Therefore, there is a discrepancy of around 15° between the lattice and phenomenological results. An improved lattice calculation of θ_{3P_1} will be desired.

In short, we conclude that $\theta_{3P_1} \approx 23^\circ$ and $\theta_{1P_1} \approx 28^\circ$ are strongly preferred as they are close to the ideal mixing and much favored by the phenomenological analysis. This in turn implies the preference of $\theta_{K_1} \sim 34^\circ$ over 57° .

V. CONCLUSIONS

Various phenomenological studies indicate that the K_{1A} - K_{1B} mixing angle θ_{K_1} lies in the vicinity of 35° or 55° , but there is no consensus as to whether this angle is greater or less than 45° . The values of the $f_1(1285)$ - $f_1(1420)$ and $h_1(1170)$ - $h_1(1380)$ mixing angles θ_{3P_1} and θ_{1P_1} , respectively, are summarized in Table I for some representative θ_{K_1} as they depend on the masses of K_{1A} and K_{1B} , which in turn depend on the mixing angle θ_{K_1} . The approximate decoupling of the light $q\bar{q}$ state from the heavier $s\bar{s}$ state, which is empirically successful for vector, tensor and 3^{--} mesons, should be also valid for other isoscalar mesons except for the pseudoscalar ones. When applying this nearly complete decoupling to axial-vector mesons, we are able to discriminate different solutions of θ_{3P_1} and θ_{1P_1} and pick up $\theta_{K_1} \sim 35^\circ$ over 55° . For $\theta_{K_1} \sim 55^\circ$, the predicted θ_{1P_1} disagrees sharply with the recent lattice calculation and the large $s\bar{s}$ content of $h_1(1170)$ and $q\bar{q}$ content of $h_1(1380)$ cannot explain the observation of their strong decays. Therefore, we conclude that θ_{K_1} is smaller than 45° and that $\theta_{3P_1} \sim 23^\circ$ and $\theta_{1P_1} \sim 28^\circ$.

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⁶ This is different from the relation $\theta = \alpha - 54.74^\circ$ used in [24]. Note that the mixing angle $\theta_{3P_1} = (56^{+4}_{-5})^\circ$ obtained in [14] cannot be deduced from $\alpha_{3P_1} = (21 \pm 5)^\circ$ quoted in [24] through the latter relation.

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